

Indirect determination of the Kugo-Ojima function from lattice data

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Abstract

We study the structure and non-perturbative properties of a special Green's function, $u(q^2)$, whose infrared behavior has traditionally served as the standard criterion for the realization of the Kugo-Ojima confinement mechanism. It turns out that, in the Landau gauge, $u(q^2)$ can be determined from a dynamical equation, whose main ingredients are the gluon propagator and the ghost dressing function, integrated over all physical momenta. Using as input for these two (infrared finite) quantities recent lattice data, we obtain an indirect determination of $u(q^2)$. The results of this mixed procedure are in excellent agreement with those found previously on the lattice, through a direct simulation of this function. Most importantly, in the deep infrared the function deviates considerably from the value associated with the realization of the aforementioned confinement scenario. In addition, the dependence of $u(q^2)$, and especially of its value at the origin, on the renormalization point is clearly established. Some of the possible implications of these results are briefly discussed.

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I. INTRODUCTION

The problem of quark confinement and gluon screening is of central importance in QCD, and a large body of work has been dedicated to its understanding [1]. Some of the most widely explored mechanisms attempting to explain how quarks confine make concrete predictions about the non-perturbative behavior of the fundamental Green's functions of the theory. For example, a central ingredient in the center vortex picture of confinement put forth by Cornwall is the dynamical generation of a gluon mass [2] through the well-known Schwinger mechanism [3, 4, 5], implemented within the pinch technique (PT) framework [2, 6, 7]. In addition to taming the infrared divergences intrinsic to perturbation theory (Landau pole), this mass gives rise to a low energy effective theory [8] which supports quantum solitons (center vortices), not present in the massless theory, whose condensation furnishes an area law to the fundamental representation Wilson loop, thus confining quarks [9]. On the other hand, the adjoint potential shows a roughly linear regime followed by string breaking when the potential energy is about $2m$, where m is the induced mass of the gluon [10, 11] corresponding to gluon screening [12, 13]. At the level of the two fundamental Green's functions of the theory, namely the gluon and ghost propagators, the predictions of the above picture are very definite: the gluon propagator is infrared finite (due to the generation of the gluon mass [15], whose phenomenological value has been delimited in Ref. [14]), while, as has been shown recently [15, 16], in the Landau gauge the ghost remains massless, but with a finite dressing function (due to the saturation produced by the gluon mass) [17].

An entirely different set of predictions is obtained within the Kugo-Ojima (KO) scenario, which also establishes a highly non-trivial link between confinement and the infrared behavior of some of the most fundamental Green's functions of QCD. In the KO confinement picture (in covariant gauges), the absence of colored asymptotic states from the physical spectrum of the theory is due to the so-called quartet mechanism [18]. A sufficient condition for the realization of this mechanism (and the meaningful definition of a conserved BRST charge) is that a certain correlation function, to be denoted by $u(q^2)$, defined in Eq. (2.21), should satisfy the condition $u(0) = -1$ [19]. In addition, as first noted by Kugo [20], in the Landau gauge, $u(0)$ is related to the infrared behavior of the ghost dressing function $F(q^2)$ [see Eq. (2.7)] through the identity $F^{-1}(0) = 1 + u(0)$. Therefore, the KO confinement scenario predicts a divergent ghost dressing function, and vice-versa. Interestingly enough, the

same prediction about $F^{-1}(0)$ is obtained when implementing the Gribov-Zwanziger (GZ) horizon condition [21, 22]: in the IR region the ghost propagator diverges more rapidly than at tree-level [23]. Furthermore, it has been also argued that the Landau gauge gluon propagator should vanish in the same limit [24]. This alleged connection between confinement, the horizon condition, and an infrared “enhanced” ghost dressing function has served as the theoretical cornerstone of the so called “ghost-dominance” picture of QCD [25].

Turns out that recent large volume lattice simulations [26, 27] appear to be at odds with the original KO and GZ pictures described above, at least as far as their predictions about the infrared behavior of the Green’s functions are concerned [28]. Specifically, various lattice studies, both in $SU(2)$ and $SU(3)$, find (in the Landau gauge) an infrared finite gluon propagator [29] and an infrared finite (“non-enhanced”) ghost dressing function. Evidently, if taken at face value [30], these results furnish strong support for the PT picture of dynamical gluon mass generation and the ensuing confinement mechanism.

It is perfectly clear that further detailed scrutiny from all possible angles must be implemented before reaching a definite conclusion on any of these issues. In this vein, it is natural to ask what one really knows about the KO function $u(q^2)$. Turns out that $u(q^2)$ has been studied directly on the lattice by Sternbeck [31], using the field-theoretic definition of $u(q^2)$ appearing in the KO formulation. As can be plainly seen from Sternbeck’s result (reproduced for convenience in Fig. 8 of this article), $u(0)$ deviates appreciably from its KO value of -1 ; specifically, the function $u(q^2)$ saturates in the deep infrared around approximately -0.6 . Interestingly enough, in a recent article [32] Kondo gave a simple derivation of this same value, after appropriately modifying the KO construction in order to self-consistently accommodate the GZ horizon condition.

Quite remarkably, in the (background) Landau gauge [33] the KO function coincides with a certain auxiliary function, usually denoted by $G(q^2)$, which constitutes a crucial ingredient in the modern formulation of the PT by means of the Batalin-Vilkovisky (BV) quantization formalism [34]. Specifically, $G(q^2)$ is the form-factor multiplying $g_{\mu\nu}$ in the Lorentz decomposition of a special Green’s function, denoted by $\Lambda_{\mu\nu}(q)$, which enters in all “background-quantum” identities [35, 36], *i.e.*, the infinite tower of non-trivial relations connecting the Green’s functions of the background field method (BFM) [37] to the conventional ones (e.g. R_ξ gauges). Notice also that $G(q^2)$ plays a prominent role in the new Schwinger-Dyson (SD) equations derived within the PT framework [38], which, due to the

special properties of the Green's functions involved, can be truncated in a manifestly gauge invariant way [39].

As has been shown in a recent article [40], one may derive a dynamical (SD-like) equation for $G(q^2)$, which, under mild assumptions, allows one to reconstruct $G(q^2)$ from the knowledge of the gluon and ghost propagators. Specifically, $G(q^2)$ is determined by integrating over all virtual momenta (k) a kernel involving the product $\Delta(k)F(k+q)$. We emphasize that the aforementioned dynamical equation is not a simple relation of several Green's functions at some special isolated point; instead, the value obtained for $G(q^2)$ at any point (such as $q^2 = 0$) must be compatible with the behavior of F and Δ in the entire range of their physical (euclidean) momenta. In particular, one must know their behavior not only in the IR, but also in the intermediate region of momenta (0.3-3 GeV), which appears rather difficult to obtain from SD studies [41]. This feature is very powerful, because it probes the details of the fundamental Green's functions over an extended range of momenta, rather than just a single point.

In the present work, we use the available lattice data on the gluon and ghost propagator as input into the aforementioned dynamical equation, thus obtaining an indirect determination of $G(q^2)$ in the entire range of available lattice momenta. Given the Landau gauge coincidence between $G(q^2)$ and $u(q^2)$, this procedure automatically determines the KO function as well. This, in turn, permits us to obtain the value of the KO parameter $u(0)$, as well as the GZ horizon function, and study their dependence on the renormalization point μ . Our analysis reveals an impressive self-consistency between the various ingredients entering into the calculation. In particular, the results obtained through our combined method (SD using lattice data as input) are in excellent agreement with those of [31], obtained through a direct simulation of the KO function.

The paper is organized as follows. In Section II we briefly introduce the BV framework for $SU(N)$ Yang-Mills theories, where the function $\Lambda_{\mu\nu}(q)$ appears naturally. Next, we review a number of relations where this function plays a key role: (i) the background-quantum identity relating the conventional and the BFM gluon propagators; (ii) the relation between the ghost dressing function $F(q^2)$ and the $\Lambda_{\mu\nu}(q)$ form factors $G(q^2)$ and $L(q^2)$; (iii) we establish the crucial equality $u(q^2) = G(q^2)$; (iv) the relation with the GZ horizon function. The central results of this article are presented in Section III. Specifically, the Lorentz decomposition of $\Lambda_{\mu\nu}(q)$ gives rise to two form-factors, the $G(q^2)$, which in the

previous section has been identified with the KO function $u(q^2)$, and the $L(q^2)$, which has the particular property of vanishing in the deep IR. After establishing the dynamical equations governing $G(q^2)$ and $L(q^2)$, we use the recent lattice data for the gluon and ghost propagators as input in these equations. In addition to the equations for $G(q^2)$ and $L(q^2)$, we consider the SD equation for the ghost, which is calibrated in order to be numerically compatible with the lattice data (at an impressive precision) simply by adjusting the gauge coupling to values that are slightly above the standard two-loop MOM prediction; the obtained value of the coupling is then used into the equations for $G(q^2)$ and $L(q^2)$. We use the multiplicative renormalizability of the gluon and ghost propagators in order to rescale the lattice data to different values of the renormalization point. Even though this procedure has an intrinsic limitation set by the relatively short reach of the available data into the UV, it amply demonstrates that $u(q^2)$ depends non-trivially on μ , in excellent agreement with the observation established in [31]. Finally, in Section IV we present our conclusions.

II. CONNECTING THE KUGO-OJIMA AND G FUNCTIONS

As already mentioned in the Introduction, in the Landau gauge the KO function $u(q^2)$ may be shown to be identical to the function $G(q^2)$, which appears in several formal contexts. In this section we first formulate Yang-Mills theories in the BV framework, which allows the derivation of a tower of identities, whose common ingredient is the function $G(q^2)$. Then we will show why $u(q^2) = G(q^2)$, and will review the connection between $u(q^2)$ and the GZ horizon. This main purpose of this section is to serve as a reminder and to bring together various seemingly disjoint pieces of information. For specific details on each topic the reader is referred to the corresponding extensive literature.

A. Batalin-Vilkoviski formalism

In the BV formulation of Yang-Mills theories [34], one starts by introducing certain sources (called anti-fields in what follows) that describe the renormalization of composite operators; the latter class of operator is in fact bound to appear in such theories due to the non-linearity of the BRST transformation of the elementary fields. In much the same way, the quantization of the theory in a background field type of gauge requires, in addition to

the aforementioned anti-fields, the introduction of new sources which couple to the BRST variation of the background fields [35]. These sources are sufficient for implementing the full set of symmetries of a non-Abelian theory at the quantum level, and in the case of quarkless $SU(N)$ QCD, lead to the master equation

$$\int d^4x \left[\frac{\delta\Gamma}{\delta A_\mu^{*m}} \frac{\delta\Gamma}{\delta A_\mu^m} + \frac{\delta\Gamma}{\delta c^m} \frac{\delta\Gamma}{\delta \bar{c}^m} + B^m \frac{\delta\Gamma}{\delta \bar{c}^m} + \Omega_\mu^m \left(\frac{\delta\Gamma}{\delta \hat{A}_\mu^m} - \frac{\delta\Gamma}{\delta A_\mu^m} \right) \right] = 0. \quad (2.1)$$

In the formula above, Γ is the effective action, A^* and c^* the gluon and ghost anti-fields, \hat{A} is the gluon background field, and Ω the corresponding background source; finally B denotes the Nakanishi-Lautrup multiplier for the gauge fixing condition.

To determine the complete algebraic structure of the theory we need two additional equations. The first one is the Faddeev-Popov equation, that controls the result of the contraction of an anti-field leg with the corresponding momenta. In position space, it reads

$$\frac{\delta\Gamma}{\delta \bar{c}^m} + \left(\hat{\mathcal{D}}^\mu \frac{\delta\Gamma}{\delta A_\mu^*} \right)^m - (\mathcal{D}^\mu \Omega_\mu)^m = 0, \quad (2.2)$$

where $(\mathcal{D}^\mu \Phi)^m = \partial^\mu \Phi^m + g f^{mnr} A_\mu^n \Phi^r$ [in the case of $(\hat{\mathcal{D}}^\mu \Phi)^m$ replace the gluon field A with a background gluon field \hat{A}]. The second one is the anti-ghost equation formulated in the background field Landau gauge, which reads [33]

$$\frac{\delta\Gamma}{\delta c^m} - \left(\hat{\mathcal{D}}^\mu \frac{\delta\Gamma}{\delta \Omega_\mu} \right)^m - (\mathcal{D}^\mu A_\mu^*)^m - f^{mnr} c^{*n} c^r + f^{mnr} \frac{\delta\Gamma}{\delta B^n} \bar{c}^r = 0, \quad (2.3)$$

This equation fully constrains the dynamics of the ghost field c , and implies that the latter will not get an independent renormalization constant. The local form of the anti-ghost equation (2.3) is only valid when choosing the background Landau gauge condition $(\hat{\mathcal{D}}^\mu A_\mu)^m = 0$; in the usual Landau gauge, $\partial^\mu A_\mu^m = 0$, an integrated version of this equation is available. In fact, even though the results that follow will be derived for convenience in the background Landau gauge, they are valid also in the conventional Landau gauge of the R_ξ .

Now, differentiation of the functional (2.1) with respect to a combination of fields containing at least one ghost field or two ghost fields and one anti-field (and setting the fields and sources to zero afterwards) will provide the Slavnov-Taylor identities of the theory. Differentiation with respect to a background source and background or quantum fields will provide, instead, the so called background-quantum identities [35, 36], which relate 1PI Green's functions involving background fields with those involving quantum fields. Finally,

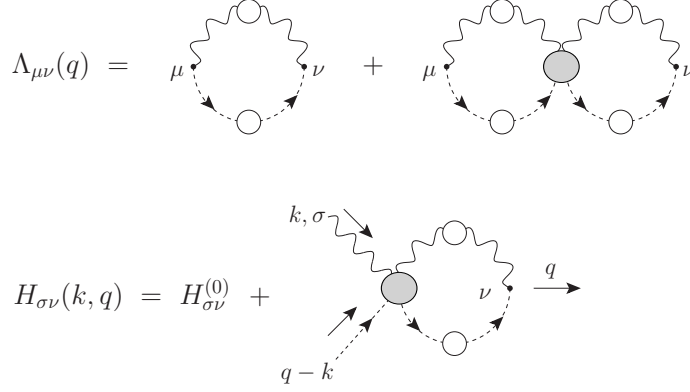


FIG. 1: Diagrammatic representation of the functions Λ and H .

differentiation of (2.2) and (2.3) with respect to fields and anti-fields or background sources give rise to relation among the different auxiliary ghost functions appearing in the theory.

The important point is that, when carrying out these differentiations, the following function appears (Fig. 1)

$$\begin{aligned}
 i\Lambda_{\mu\nu}(q) &= \Gamma_{\Omega_\mu A_\nu^*}(q) \\
 &= g^2 C_A \int_k H_{\mu\rho}^{(0)} D(k+q) \Delta^{\rho\sigma}(k) H_{\sigma\nu}(k, q), \\
 &= ig_{\mu\nu} G(q^2) + i \frac{q_\mu q_\nu}{q^2} L(q^2),
 \end{aligned} \tag{2.4}$$

and (in d -dimensions)

$$G(q^2) = \frac{1}{(d-1)q^2} (q^2 \Lambda_\mu^\mu - q^\mu q^\nu \Lambda_{\mu\nu}), \quad L(q^2) = \frac{1}{(d-1)q^2} (dq^\mu q^\nu \Lambda_{\mu\nu} - q^2 \Lambda_\mu^\mu). \tag{2.5}$$

In the equations above, the color factor δ^{mn} has been factored out (as always in what follows), C_A represents the Casimir eigenvalue of the adjoint representation [$C_A = N$ for $SU(N)$], and $\int_k \equiv \mu^{2\epsilon} (2\pi)^{-d} \int d^d k$, with $d = 4 - \epsilon$ the dimension of space-time. $\Delta_{\mu\nu}$ and D represents the gluon and ghost propagator respectively, defined as

$$\Delta_{\mu\nu}(q) = -i \left[P_{\mu\nu}(q) \Delta(q^2) + \xi \frac{q_\mu q_\nu}{q^4} \right], \tag{2.6}$$

$$D(q^2) = \frac{iF(q^2)}{q^2}, \tag{2.7}$$

where ξ denotes the gauge-fixing parameter, and $P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$ is the usual transverse projector; notice that $\Delta^{-1}(q^2) = q^2 + i\Pi(q^2)$, with $\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q^2)$ the gluon self-energy. $F(q^2)$ is the so called ghost dressing function. Finally, the function $H_{\mu\nu}(k, q)$

(see Fig. 1 again) is in fact a familiar object, since it appears in the all-order Slavnov-Taylor identity satisfied by the standard three-gluon vertex [42]

$$\begin{aligned} q^\alpha \Gamma_{\alpha\mu\nu}(q, k_1, k_2) &= iF(q^2) \Delta^{-1}(k_1^2) P_\mu^\rho(k_1) H_{\nu\rho}(k_2, k_1) \\ &\quad - iF(q^2) \Delta^{-1}(k_2^2) P_\nu^\rho(k_2) H_{\mu\rho}(k_1, k_2). \end{aligned} \quad (2.8)$$

It is also related to the full gluon-ghost vertex $\mathbf{\Gamma}_\mu(k, q)$ by the identity

$$q^\nu H_{\mu\nu}(k, q) = -i\mathbf{\Gamma}_\mu(k, q). \quad (2.9)$$

At tree-level, $H_{\mu\nu}^{(0)} = ig_{\mu\nu}$ and $\mathbf{\Gamma}_\mu^{(0)}(k, q) = \Gamma_\mu(k, q) = -q_\mu$.

B. Background-quantum identities

The first identities where the function $\Lambda_{\mu\nu}$ appears are the so-called background-quantum identities, *i.e.*, the infinite tower of non-trivial relations connecting the BFM Green's functions to the conventional ones [35, 36]. Consider, for example, the result of differentiating the functional (2.1) with respect to a background source and a background gluon, on the one hand, and a background source and a quantum gluon, on the other, one obtains two equations,

$$\begin{aligned} \Gamma_{\hat{A}_\mu A_\nu}(q) &= [g_{\mu\rho} + \Lambda_{\mu\rho}(q)] \Gamma_{A^\rho A_\nu}(q), \\ \Gamma_{\hat{A}_\mu \hat{A}_\nu}(q) &= [g_{\mu\rho} + \Lambda_{\mu\rho}(q)] \Gamma_{A^\rho \hat{A}_\nu}(q). \end{aligned} \quad (2.10)$$

Using the transversality of the gluon two-point function, these two equations can then be appropriately combined to yield the important identity

$$\Gamma_{\hat{A}_\mu \hat{A}_\nu}(q) = [1 + G(q^2)]^2 \Gamma_{A_\mu A_\nu}(q), \quad (2.11)$$

or, in terms of propagators

$$\hat{\Delta}^{-1}(q^2) = [1 + G(q^2)]^2 \Delta^{-1}(q^2). \quad (2.12)$$

The quantity $\hat{\Delta}(q^2)$ appearing on the left-hand side of the above equation captures the running of the QCD β function, exactly as happens with the QED vacuum polarization; this is a fundamental property of the BFM gluon self-energy, valid for every value of the

(quantum) gauge-fixing parameter [37]. This can be easily checked to lowest order, where Eqs. (2.4) and (2.5) give (in the Landau gauge)

$$\begin{aligned} 1 + G(q^2) &= 1 + \frac{9}{4} \frac{C_A g^2}{48\pi^2} \ln\left(\frac{q^2}{\mu^2}\right), \\ \Delta^{-1}(q^2) &= q^2 \left[1 + \frac{13}{2} \frac{C_A g^2}{48\pi^2} \ln\left(\frac{q^2}{\mu^2}\right) \right], \end{aligned} \quad (2.13)$$

and thus

$$\widehat{\Delta}^{-1}(q^2) = q^2 \left[1 + b g^2 \ln\left(\frac{q^2}{\mu^2}\right) \right]. \quad (2.14)$$

where $b = 11C_A/48\pi^2$ is the first coefficient in the QCD β function. Eq. (2.12) plays a central role in the derivation of a new set of SDEs [38] that can be truncated in a manifestly gauge invariant way [39].

Let us conclude this subsection by noticing that in more general identities the function $L(q^2)$ is also relevant. Consider *e.g.*, the identity relating the three-gluon proper vertices. One then has

$$\begin{aligned} \Gamma_{\widehat{A}_\mu A_\alpha A_\beta}(k_1, k_2) &= [g_\mu^\nu + \Lambda_\mu^\nu(q)] \Gamma_{A_\nu A_\alpha A_\beta}(k_1, k_2) + \cdots \\ &= [1 + G(q^2)] \Gamma_{A_\mu A_\alpha A_\beta}(k_1, k_2) + \frac{q_\mu q^\nu}{q^2} L(q^2) \Gamma_{A_\nu A_\alpha A_\beta}(k_1, k_2) + \cdots, \end{aligned} \quad (2.15)$$

where the omitted terms involve other auxiliary Green's functions (see [35, 36]), irrelevant to our discussion.

C. Two-point ghost sector

Let us now consider the two-point functions. Differentiating the ghost equation (2.2) with respect to a ghost field and a background source we get the relations

$$\begin{aligned} \Gamma_{c\bar{c}}(q) &= -iq^\nu \Gamma_{cA_\nu^*}(q), \\ \Gamma_{\bar{c}\Omega_\mu}(q) &= q_\mu + q^\nu \Lambda_{\mu\nu}(q). \end{aligned} \quad (2.16)$$

On the other hand, differentiating the anti-ghost equation (2.3) with respect to a gluon anti-field and an anti-ghost, one gets

$$\begin{aligned} \Gamma_{cA_\nu^*}(q) &= q_\nu + q^\mu \Lambda_{\mu\nu}(q), \\ \Gamma_{c\bar{c}}(q) &= -iq^\mu \Gamma_{\bar{c}\Omega_\mu}(q). \end{aligned} \quad (2.17)$$

$$-G_{\mu\nu}^{mn}(q) = \text{Diagram 1} + \text{Diagram 2}$$

Diagram 1: A horizontal line with a black circle in the middle. The left part is labeled Ω_μ^m and the right part is labeled A_ν^{*m} .

Diagram 2: A horizontal line with three elements: a black circle, a white circle, and a black circle. The left part is labeled Ω_μ^m , the middle white circle is labeled \bar{c}^s , and the right part is labeled A_ν^{*m} . Arrows point from the white circle to the two black circles.

FIG. 2: Connected components contributing to the function $G_{\mu\nu}^{mn}(q)$.

Next, contracting the first equation in (2.17) with q^ν , and making use of the first equation in (2.16), we see that the dynamics of the ghost sector is entirely captured by the $\Lambda_{\mu\nu}$ auxiliary function, since

$$i\Gamma_{c\bar{c}}(q) = q^2 + q^\mu q^\nu \Lambda_{\mu\nu}(q). \quad (2.18)$$

Introducing the Lorentz decompositions

$$\Gamma_{cA_\mu^*}(q) = q_\mu C(q^2), \quad \Gamma_{\bar{c}\Omega_u}(q) = q_\mu E(q^2), \quad (2.19)$$

we find that Eq. (2.18) together with the last equation of (2.16) and (2.17) give the identities [20, 33]

$$\begin{aligned} C(q^2) &= E(q^2) = F^{-1}(q^2), \\ F^{-1}(q^2) &= 1 + G(q^2) + L(q^2). \end{aligned} \quad (2.20)$$

Finally, recalling that the dimension of the gluon anti-field A^* is three, while the dimension of the Ω source is one, power counting shows that (i) all functions appearing in Eqs. (2.16) and (2.17) are divergent, and (ii) the divergent part of $\Lambda_{\mu\nu}(q)$ can be proportional to $g_{\mu\nu}$ only [33, 40].

D. The (background) Landau gauge equality between $u(q^2)$ and $G(q^2)$

A crucial ingredient to our analysis is the equality between the KO function and the $G(q^2)$, in the (background) Landau gauge [33]. To see this, we start with the following (Euclidean) two-point function of composite operators

$$\int d^4x \, e^{-iq \cdot (x-y)} \langle T [(\mathcal{D}_\mu c)_x^m (f^{nrs} A_\nu^n \bar{c}^s)_y] \rangle = P_{\mu\nu}(q) \delta^{mn} u(q^2), \quad (2.21)$$

which, due to the identity

$$\int d^4x \, e^{-iq \cdot (x-y)} \langle T [(\mathcal{D}_\mu c)_x^m \bar{c}^n] \rangle = -i \frac{q_\mu}{q^2} \delta^{mn} \quad (2.22)$$

can be related to the function $\langle [(\mathcal{D}_\mu c)_x^m (\mathcal{D}_\mu \bar{c})_y^n] \rangle$, through

$$\int d^4x e^{-iq \cdot (x-y)} \langle T [(\mathcal{D}_\mu c)_x^m (\mathcal{D}_\mu \bar{c})_y^n] \rangle = -\frac{q_\mu q_\nu}{q^2} \delta^{mn} + P_{\mu\nu}(q) \delta^{mn} u(q^2). \quad (2.23)$$

On the other hand, observe that in the background Landau gauge the function appearing on the lhs of the above equation is precisely given by

$$-\mathcal{G}_{\mu\nu}^{mn}(q) = \frac{\delta^2 W}{\delta \Omega_\mu^m \delta A_\nu^{*n}}, \quad (2.24)$$

where W is the generator of the connected Green's functions, and the two connected diagrams contributing to $\mathcal{G}_{\mu\nu}$ are shown in Fig. 2. Factoring out the color structure and making use of the identities (2.20) one has

$$\begin{aligned} -i\mathcal{G}_{\mu\nu}(q) &= \Lambda_{\mu\nu}(q) + \Gamma_{\Omega_\mu \bar{c}}(q) D(q^2) \Gamma_{A_\nu^* c}(q) \\ &= -\frac{q_\mu q_\nu}{q^2} + P_{\mu\nu}(q) G(q^2). \end{aligned} \quad (2.25)$$

Passing to the Euclidean formulation, and comparing with Eq. (2.23), we then arrive at the important equality

$$u(q^2) = G(q^2). \quad (2.26)$$

Then, the usual KO confinement criterion may be equivalently cast in the form: $1+G(0) = 0$. Evidently, if $L(0) = 0$ [see discussion after Eq. (3.6)], then from the identity (2.20) follows that if the KO criterion is satisfied then the ghost dressing function diverges in the IR.

E. Gribov-Zwanziger horizon

In order to avoid Gribov copies [21], in the GZ formulation of Yang-Mills theories the partition function assumes the form (in d -dimensional Euclidean space) [22]

$$Z_\gamma = \int [dA] \delta(\partial^\mu A_\mu) \text{Det} M \exp \left\{ -S_{\text{YM}} + \gamma \int d^d x h(x) \right\}, \quad (2.27)$$

where S_{YM} is the Yang-Mills action, $M = -\partial_\mu \mathcal{D}^\mu$ is the Faddeev-Popov operator, and the functional $h(x) = h[A](x)$ is the so-called GZ horizon function given by

$$h(x) = - \int d^d y g f^{amr} A_\mu^m(x) (M^{-1})^{rs}(x, y) g f^{asn} A_\nu^n(y); \quad (2.28)$$

thus, the action corresponding to the partition function above clearly contains a non-local term. The GZ parameter, γ , is determined through the so-called *horizon condition*, which for $SU(N)$ assumes the form

$$\langle h(x) \rangle_\gamma = d(N^2 - 1). \quad (2.29)$$

This condition can be rewritten in terms of the vev of the GZ horizon function if we integrate both sides over $d^d x$. In this way we get

$$\begin{aligned} \langle h(0) \rangle_\gamma &\equiv \frac{1}{V_d} \frac{\partial}{\partial \gamma} \ln Z_\gamma \\ &= \frac{1}{V_d} \int d^d x \langle h(x) \rangle_\gamma = d(N^2 - 1). \end{aligned} \quad (2.30)$$

On the other hand, assuming that γ is small, one can expand in powers of γ ; retaining the first order only, one gets

$$\langle h(0) \rangle_\gamma \simeq \langle h(0) \rangle_{\gamma=0} + \mathcal{O}(\gamma). \quad (2.31)$$

The right-hand side of the above equation can be related to the trace of the following Green's function (Euclidean space)

$$\mathcal{H}_{\mu\nu}^{mn}(q) = \delta^{mn} \left[P_\mu^\rho(q) + \frac{q_\mu q^\rho}{q^2} F(q^2) \right] \Lambda_{\rho\nu}(q), \quad (2.32)$$

in the limit $q^2 \rightarrow 0$ [32]. Specifically,

$$\begin{aligned} \langle h(0) \rangle_{\gamma=0} &= \frac{1}{V_d} \int d^d x \langle h(x) \rangle_{\gamma=0} = - \lim_{q^2 \rightarrow 0} \text{Tr} \{ \mathcal{H}_{\mu\nu}^{mn}(q) \} \\ &= -(N^2 - 1) \{ (d-1)G(0) + F(0) [G(0) + L(0)] \}. \end{aligned} \quad (2.33)$$

This result allows to rewrite the GZ horizon condition in terms of $G(0)$ (and therefore of the KO parameter $u(0)$); this will, in turn, restrict the allowed values of $u(0)$. In the limit of vanishing Gribov parameter, one can use the result of (2.33) to solve the horizon condition (2.29), in the approximation (2.31); If $L(0) = 0$, one finds (in $d = 4$) the following value of the KO parameter [32]

$$u(0) = G(0) = -\frac{2}{3}, \quad (2.34)$$

which is very close to that obtained directly from the lattice [31], and, as we will see in the next section, from our independent analysis.

III. EXTRACTING THE KUGO-OJIMA FUNCTION FROM THE LATTICE

In this section we study the behavior of the function $G(q^2)$ by using the available lattice data on the gluon and ghost propagators. Specifically, we will first write down the dynamical equations that govern the functions $G(q^2)$ and $L(q^2)$, which involve both $\Delta(q^2)$ and $D(q^2)$, and using as an input the lattice results for these propagators we will obtain an indirect determination of $G(q^2)$. Of course, by virtue of the fundamental equality $u(q^2) = G(q^2)$, any information on $G(q^2)$ translates automatically to the Kugo-Ojima function.

A. Dynamical equations

The dynamical equations governing $G(q^2)$ and $L(q^2)$ may be obtained directly from the defining equation (2.4), by appropriately contracting it and taking its trace. Specifically, from Eq. (2.4) one has (in d -dimensions) [40]

$$\begin{aligned} G(q^2) &= \frac{g^2 C_A}{d-1} \left[\int_k \Delta^{\rho\sigma}(k) H_{\sigma\rho}(k, q) D(k+q) + i \frac{1}{q^2} \int_k q^\rho \Delta_{\rho\sigma}(k) \Gamma^\sigma(k, q) D(k+q) \right], \\ L(q^2) &= -\frac{g^2 C_A}{d-1} \left[i \frac{d}{q^2} \int_k q^\rho \Delta_{\rho\sigma}(k) \Gamma^\sigma(k, q) D(k+q) + \int_k \Delta^{\rho\sigma}(k) H_{\sigma\rho}(k, q) D(k+q) \right]. \end{aligned} \quad (3.1)$$

Adding the above equations by parts, and employing (2.9), one may easily demonstrate [40], in the Landau gauge, the validity of the second identity in Eq. (2.20), given that the standard SD equation for the ghost propagator (Fig. 3) reads

$$iD^{-1}(q^2) = q^2 + ig^2 C_A \int_k \Gamma^\mu \Delta_{\mu\nu}(k) \Gamma^\nu(k, q) D(q+k). \quad (3.2)$$

The two equations in (3.1) involve five basic ingredients: the two-point functions $\Delta(q^2)$ and $D(q^2)$, the vertex functions $\Gamma_\mu(k, q)$ and $H_{\mu\nu}(q, k)$, and, eventually, the value of the (renormalized) coupling g^2 , at different renormalization points. Knowledge (direct or indirect) of these ingredients (for example from the lattice) would, in turn, determine fully the functions $G(q^2)$ and $L(q^2)$.

As far as the vertex functions $\Gamma_\mu(k, q)$ and $H_{\mu\nu}(k, q)$ are concerned, due to the transversality of the gluon propagator, the only relevant form factors are

$$\begin{aligned} -\Gamma_\mu(k, q) &= B_1(k, q) q_\mu, \\ -iH_{\mu\nu}(k, q) &= A_1(k, q) g_{\mu\nu} + A_2(k, q) q_\mu q_\nu. \end{aligned} \quad (3.3)$$

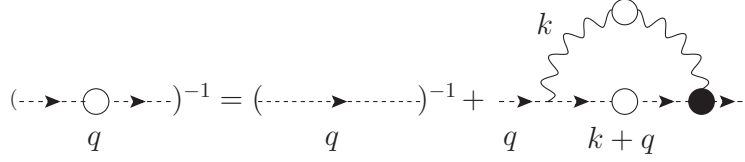


FIG. 3: The SDE satisfied by the ghost propagator.

In order to study the relevant equations further, we will approximate the form factors above with their tree-level values, *i.e.*, $A_1(k, q) = B_1(k, q) = 1$, and $A_2(k, q) = 0$; according to lattice studies [43], this appears to be a very good approximation. Then, the equations in (3.1) become

$$\begin{aligned} G(q^2) &= \frac{g^2 C_A}{d-1} \int_k \left[(d-2) + \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k+q), \\ L(q^2) &= \frac{g^2 C_A}{d-1} \int_k \left[1 - d \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k+q). \end{aligned} \quad (3.4)$$

Now, as discussed in detail in [40] the (unrenormalized) equations in (3.4) must be properly renormalized, *i.e.*, in such a way as to preserve the validity of the (BRST-induced) second identity in (2.20). This finally amounts to renormalizing these equations through multiplication by Z_c , the renormalization constant of the ghost propagator. Specifically, after introducing

$$\begin{aligned} \Delta(q^2; \mu^2) &= Z_A^{-1}(\mu^2) \Delta_0(q^2), \\ F(q^2; \mu^2) &= Z_c^{-1}(\mu^2) F_0(q^2), \\ g(\mu^2) &= Z_g^{-1}(\mu^2) g_0, \end{aligned} \quad (3.5)$$

we obtain in the Euclidean space, after setting $q^2 = x$, $k^2 = y$ and $\alpha_s = g^2/4\pi$, and implementing the standard angular approximation,

$$\begin{aligned} 1 + G(x) &= Z_c - \frac{\alpha_s C_A}{16\pi} \left[\frac{F(x)}{x} \int_0^x dy \, y \left(3 + \frac{y}{3x} \right) \Delta(y) + \int_x^\infty dy \left(3 + \frac{x}{3y} \right) \Delta(y) F(y) \right], \\ L(x) &= \frac{\alpha_s C_A}{12\pi} \left[\frac{F(x)}{x^2} \int_0^x dy \, y^2 \Delta(y) + x \int_x^\infty dy \frac{\Delta(y) F(y)}{y} \right]. \end{aligned} \quad (3.6)$$

From this last equation it is easy to see (*e.g.*, by means of the change of variables $y = zx$) that if Δ and F are IR finite, then $L(0) = 0$, as mentioned before [44].

After imposing the renormalization condition $F(\mu^2) = 1$, one finds that the renormaliza-

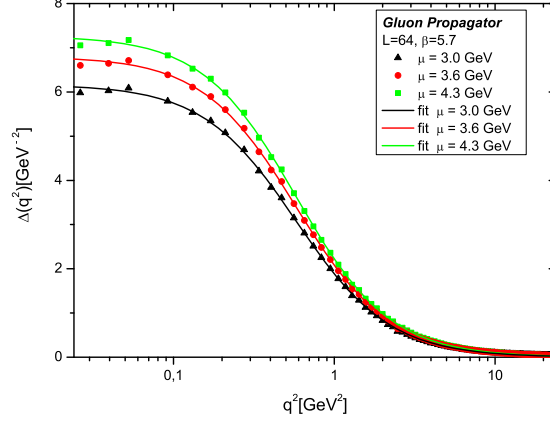


FIG. 4: Lattice results for the gluon propagator renormalized at three different renormalization points: $\mu = 3.0 \text{ GeV}$ (black curve), $\mu = 3.6 \text{ GeV}$ (red curve) and $\mu = 4.3 \text{ GeV}$ (green curve). We also show the corresponding fits using Eq.(3.10). The fitting parameters are: $a = 0.162 \text{ GeV}^2$, $b = 0.367 \text{ GeV}^{-1}$ and $c = 1.5$ ($\mu = 3.0 \text{ GeV}$); $a = 0.147 \text{ GeV}^2$, $b = 0.334 \text{ GeV}^{-1}$ and $c = 1.5$ ($\mu = 3.6 \text{ GeV}$); $a = 0.137 \text{ GeV}^2$, $b = 0.311 \text{ GeV}^{-1}$ and $c = 1.5$ ($\mu = 4.3 \text{ GeV}$).

tion constant Z_c appearing in the equation for $G(x)$ is given by

$$Z_c = 1 + \frac{\alpha_s C_A}{16\pi} \left[\frac{1}{\mu^2} \int_0^{\mu^2} dy y \left(3 - \frac{y}{\mu^2} \right) \Delta(y) + \int_{\mu^2}^{\infty} dy \left(3 - \frac{\mu^2}{y} \right) \Delta(y) F(y) \right]. \quad (3.7)$$

This renormalization constant renormalizes Eq. (3.2) as well, when the latter is evaluated under the same approximations, *i.e.*,

$$F^{-1}(x) = Z_c - \frac{\alpha_s C_A}{16\pi} \left[\frac{F(x)}{x} \int_0^x dy y \left(3 - \frac{y}{x} \right) \Delta(y) + \int_x^{\infty} dy \left(3 - \frac{x}{y} \right) \Delta(y) F(y) \right]. \quad (3.8)$$

Note that one cannot choose simultaneously the condition $G(\mu^2) = 0$ once $F(\mu^2) = 1$ has been imposed; indeed, given that $L(\mu^2) \neq 0$, such a choice would violate the identity of Eq. (2.20).

B. Numerical analysis

The starting point for our numerical analysis are the lattice results for the gluon propagator $\Delta(q^2)$ reported in [27]. In order to eventually study the dependence of the KO function on the renormalization point, we would like to obtain the lattice data at different

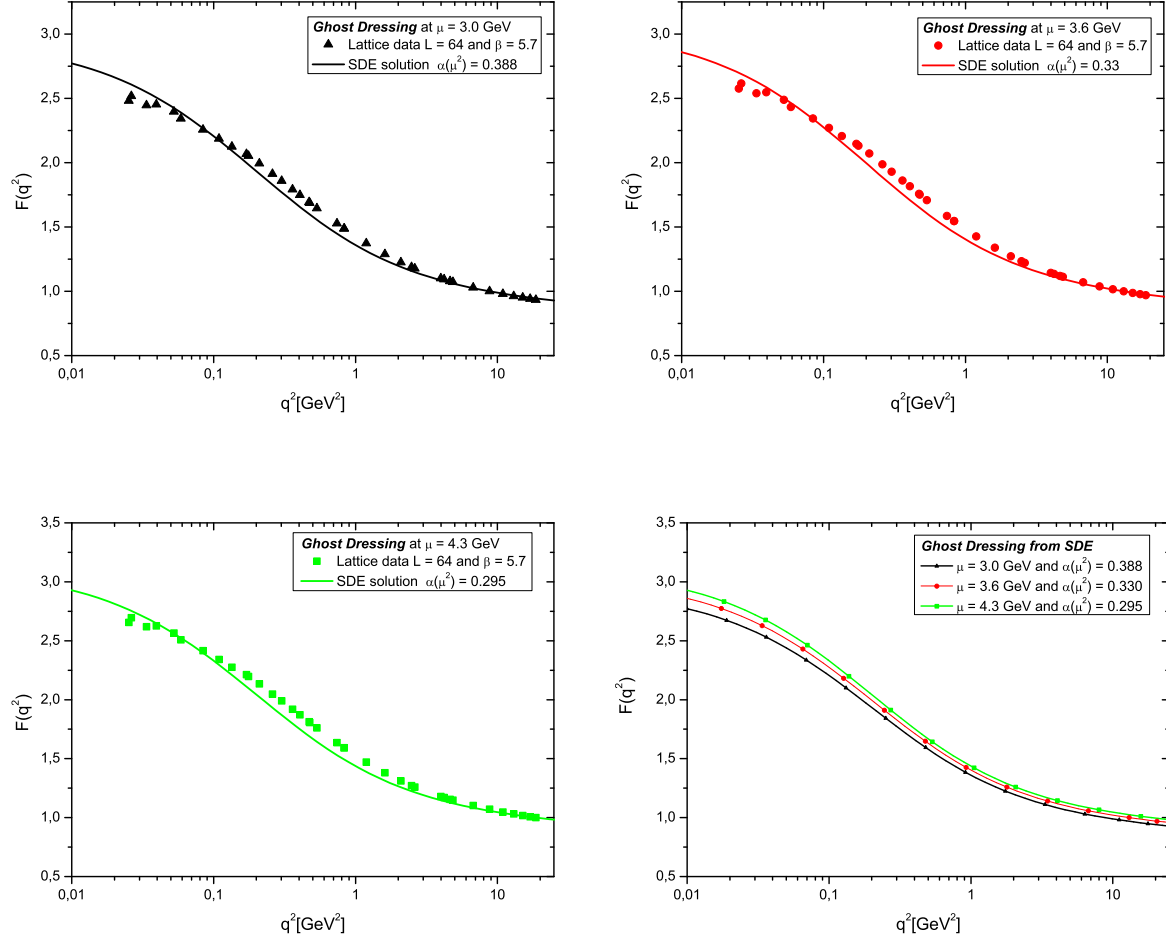


FIG. 5: *Top left panel:* The ghost dressing function $F(q^2)$ obtained from SDE (black continuous line) compared to the lattice data of [27] at $\mu = 3.0$ GeV. *Top right panel:* Same as in the previous panel but renormalized at $\mu = 3.6$ GeV. *Bottom left panel:* Same as before but renormalized at $\mu = 4.3$ GeV. *Bottom right panel:* The SDE solutions for the three different renormalization points all together.

renormalization points. Since the gluon propagator is multiplicatively renormalizable, the relation [45]

$$\Delta(q^2, \mu^2) = \frac{\Delta(q^2, \nu^2)}{\mu^2 \Delta(\mu^2, \nu^2)}, \quad (3.9)$$

can be used to connect a set of points renormalized at μ with the corresponding set renormalized at ν . Choosing the three different values $\mu = \{3.0, 3.6, 4.3\}$ GeV, we then obtain the three curves shown in Fig. 4. In the range of available momenta, a very accurate fit is

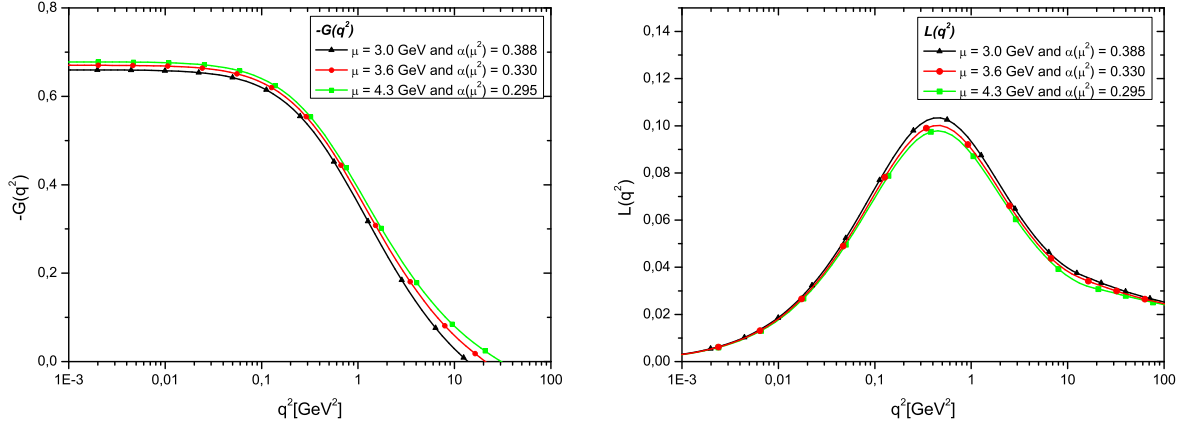


FIG. 6: *Left panel:* $-G(q^2)$ determined from Eq. (3.6), using the solutions for $\Delta(q^2)$ and $D(q^2)$ presented in Figs. 4 and 5 at the same renormalization points μ . *Right panel:* Same as in the previous panel but this time for $L(q^2)$.

provided by the expression

$$\Delta(q^2) = \frac{1}{a + bq^{2c}}, \quad (3.10)$$

as shown by the continuous line in Fig. 4 (the values of the fitting parameters a , b , and c are also reported there).

The next step is to employ the ghost SDE given in (3.8) in order to deduce the appropriate values that one must use for the gauge coupling. To that end we will follow the following steps: (i) employing once again the relation (3.9), with $\Delta \rightarrow D$, we generate from the lattice data on $F(q^2)$ reported in [27] the data sets for $F(q^2)$ corresponding to the renormalization points used previously for Δ ; (ii) using as input in (3.8) the different sets of results obtained in the previous step for Δ , we solve the integral equation (3.8) numerically, thus determining $F(q^2; \mu^2)$; (iii) the values of $\alpha_s(\mu^2)$ are fixed by demanding that the solutions obtained in (ii) match the different lattice sets generated at step (i).

The results of this procedure are displayed in Fig. 5, where we show both the comparison between the lattice data and the solutions of Eq. (3.8) (first three panels), as well as the dependence on μ^2 of these solutions (fourth panel). The couplings found are $\alpha(\mu^2) = \{0.388, 0.330, 0.295\}$ for $\mu = \{3.0, 3.6, 4.3\}$ GeV respectively. Note that the values of $\alpha_s(\mu^2)$ obtained by this procedure are about 20% higher than those found from the two-loop MOM calculation of [46].

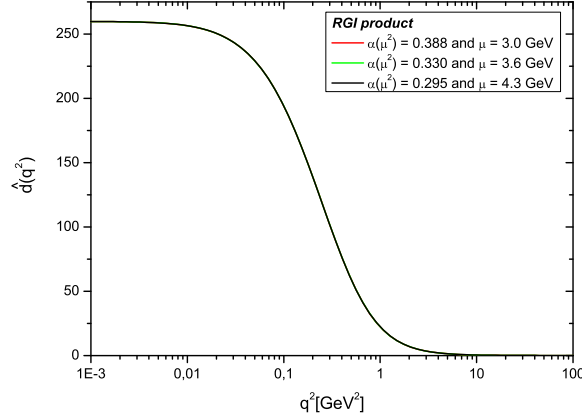


FIG. 7: The renormalization-group invariant product $\hat{d}(q^2)$ obtained combining our results for $\Delta(q^2)$ and $G(q^2)$ according to Eq. (3.11).

At this point, all necessary ingredient for determining the functions $G(q^2)$ and $L(q^2)$ are available. Substituting them into the corresponding equations given in (3.6), we obtain the solutions shown in Fig. 6. Notice that $L(q^2)$ vanishes in the deep IR, as expected.

A very stringent test of the quality of the obtained solutions can be devised by observing that, on formal grounds, the combination

$$\hat{d}(q^2) = \frac{g^2(\mu^2)\Delta(q^2; \mu^2)}{[1 + G(q^2; \mu^2)]^2}, \quad (3.11)$$

constitutes a renormalization group invariant (*i.e.*, μ -independent) quantity [47]. In Fig. 7 we plot the combination above for the three different values of μ chosen; evidently the product of $g^2(\mu^2)$, $\Delta(q^2; \mu^2)$ and $[1 + G(q^2; \mu^2)]^{-2}$ constructed from the solutions obtained is μ -independent to an extremely high degree of accuracy.

The curves plotted for $G(q^2) = u(q^2)$ (Fig. 6 left panel) should be compared with those obtained by Sternbeck [31] (reproduced in Fig. 8), where the function (2.21) was studied in terms of Monte Carlo averages, and its asymptotic behavior was inferred from the identity (2.20): in both cases, the behavior is clearly the same. Note that in [31] the extrapolation towards the zero limit was problematic, due to a lack of knowledge of the function $L(q^2)$ [there denoted by $q^2 v(q^2)$]; our analysis does not suffer from such a limitation, given that $L(q^2)$ is completely determined by its own equation.

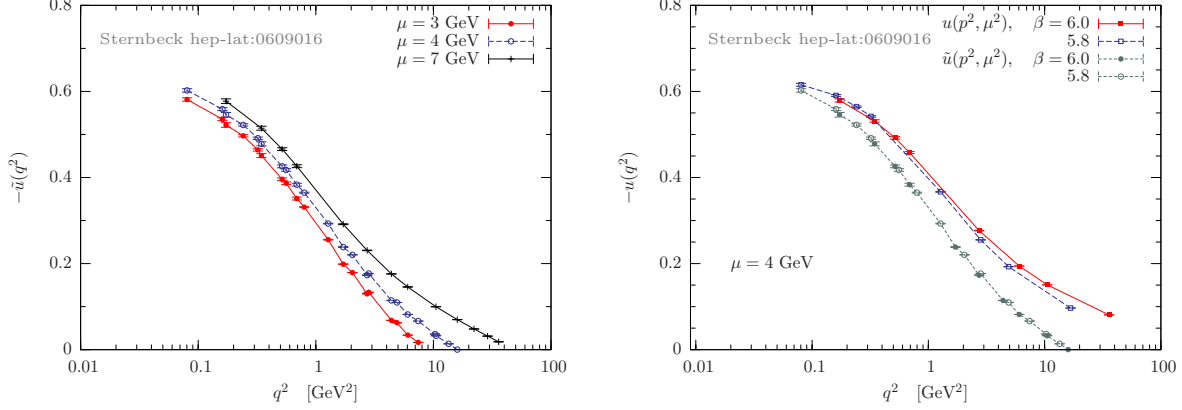


FIG. 8: The determination of the KO function obtained in [31]. *Left panel:* the function $-\tilde{u}(p^2) = 1 - F^{-1}(p^2)$, determined from Eq. (2.20) by neglecting the $L(p^2)$ component, plotted at three different renormalization points. *Right panel:* Comparison between the function $-u(p^2)$ measured directly on the lattice and its asymptotic behavior $-\tilde{u}(p^2)$.

Now, the important point to emphasize is that the function $G(q^2) = u(q^2)$ is *not* a μ -independent quantity; in fact, as we have established, its μ -dependence is exactly what is needed in order to enforce the μ -independence of the RG-invariant expression given in (3.11). In fact, its value at $q^2 = 0$, i.e. the KO parameter $u(0)$, *depends* on the renormalization point. Notice that in the case of an IR divergent ghost dressing function the possible μ -dependence would be inconsequential, since, due to the identity (2.20), $u(0) = -1$ irrespectively of the value of μ chosen. Evidently, the situation is different in the case of an IR finite ghost dressing function, since $u(0)$ acquires a non-trivial dependence on the renormalization scale. This dependence is plotted on the left panel of Fig. 9 for values of μ varying between 2.6 and 4.3 GeV (due to the limited number of UV lattice points of our data), where we see that $-G(0) = -u(0)$ varies in the interval $[0.65, 0.68]$, in good agreement with the prediction (2.34). The dependence of $u(0)$ on μ appears to be moderate, probably due to the rather narrow region of allowed μ values considered. A more detailed study with data sets extending deeper in the UV should allow one to explore the full extent of this dependence.

On the right panel Fig. 9 we plot finally the μ -dependence of the horizon function (2.33). Notice that both dependencies can be fitted with a function that is characteristic of a phase

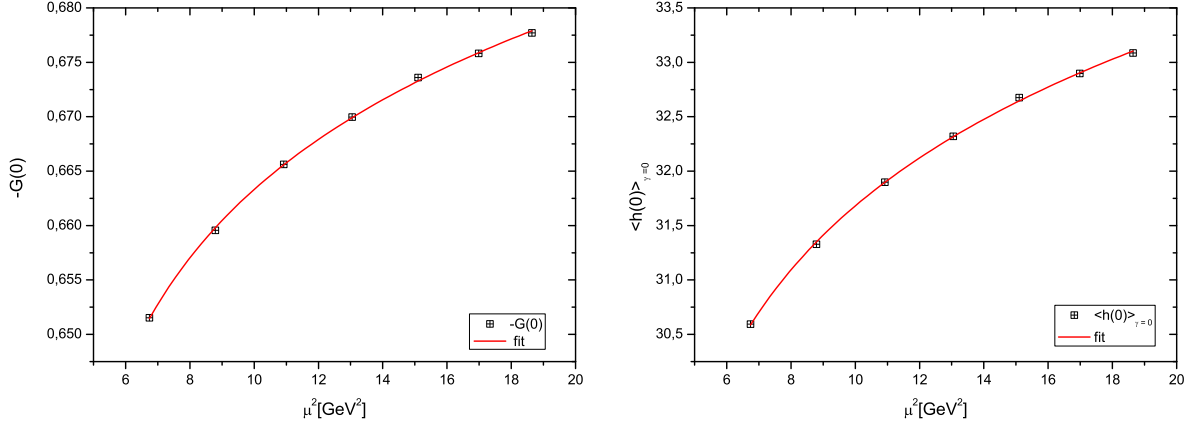


FIG. 9: *Left panel:* The dependence of the KO parameter $u(0) = G(0)$ on the renormalization point μ . *Right panel:* Same as in the previous panel but for the horizon function. In both cases the continuous red line represents the fit given by Eq. (3.13) with $a_1 = 0.63$, $b_1 = 3.57$, $c_2 = 0.025$ and $a_2 = 28.61$, $b_2 = 3.25$, $c_2 = 0.05$ for $-G(0)$ and $\langle h(0) \rangle_{\gamma=0}$ respectively.

transition, namely

$$\begin{aligned} G(0) &= -a_1(\mu^2 - b_1)^{c_1}, \\ \langle h(0) \rangle_{\gamma=0} &= -a_2(\mu^2 - b_2)^{c_2}, \end{aligned} \quad (3.12)$$

as shown by the continuous red curves appearing in Fig. 9 (the values of the fitting parameter are also reported there).

IV. CONCLUSIONS

In this work we have presented an indirect determination of the KO function from recent lattice data on the behavior of the QCD gluon and ghost propagators [26, 27] in the Landau gauge. The results obtained are in very good agreement with the original study of the same quantity presented in [31].

Of particular interest is the observed dependence of the KO function, and particular of its infrared value $u(0)$, on the renormalization point μ chosen to renormalize the theory. The μ -dependence of $u(0)$, mild as it may seem at first sight, is definitely there, as one would expect, given that the KO function $u(q^2)$ is *not* a renormalization-group invariant quantity, *i.e.*, it is not intrinsically μ -independent. In fact, the observed μ -dependence is

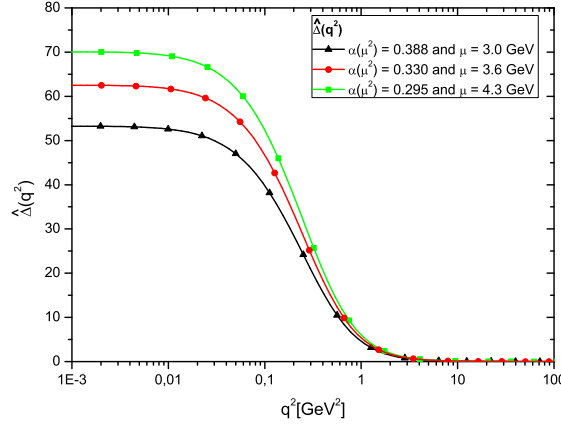


FIG. 10: The gluon propagator $\hat{\Delta}(q^2)$ of the BFM, renormalized at three different points: $\mu = 3.0 \text{ GeV}$ (black curve), $\mu = 3.6 \text{ GeV}$ (red curve) and $\mu = 4.3 \text{ GeV}$ (green curve).

really sizeable when contrasted with the impressive absence of any μ -dependence displayed by a genuinely μ -independent quantity given in Eq.(3.11) which was computed using exactly the same sets of lattice data. We hope that the present work will contribute to the study of the possible effects that renormalization may have on the quantitative predictions of the KO formalism.

Let us take a closer look at the background-quantum identity given in Eq. (2.12), which relates the conventional gluon propagator Δ with the gluon propagator $\hat{\Delta}$ of the BFM. Eq. (2.12) assumes that the corresponding gauge-fixing parameters, namely the ξ of the R_ξ and the ξ_Q used in the BFM to gauge-fix the quantum fields appearing inside the loops, are equal ($\xi = \xi_Q$). In the Landau gauge, $\xi = \xi_Q = 0$, due to the central equality of Eq. (2.26), we have that

$$u(q^2) = \sqrt{\frac{\Delta(q^2)}{\hat{\Delta}(q^2)}} - 1. \quad (4.1)$$

Interestingly enough, this simple formula expresses the KO function in terms of two gluon propagators calculated in the Landau gauge of two very distinct gauge-fixing schemes, with no direct reference to the ghost sector of the theory. This observation opens up the possibility of deducing the structure of the KO function using an entirely different, and completely novel, approach. Specifically, one may envisage a lattice simulation of $\hat{\Delta}$ [48]; then, $u(q^2)$ may be obtained from (4.1) by simply forming the ratio of the two gluon propagators. Given that $\Delta(0)$ is found to be finite on the lattice [26, 27], it is clear that, in order for the standard

KO criterion to be satisfied (*i.e.*, $u(0) = -1$), $\hat{\Delta}$ must diverge in the IR. Needless to say, we consider such a scenario highly unlikely. What is far more likely to happen, in our opinion, is to find a perfectly finite and well-behaved $\hat{\Delta}$, which in the deep IR will be about an order of magnitude larger than $\Delta(0)$, furnishing a value $u(0) \sim -0.6$, namely what we have found in our analysis. In fact, one may turn the argument around: combining the results of this article with the lattice data for Δ [26, 27], one may use (4.1) to predict the outcome of the lattice simulation for $\hat{\Delta}$; our prediction for the case of $SU(3)$ is shown in Fig. 10.

Acknowledgments

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